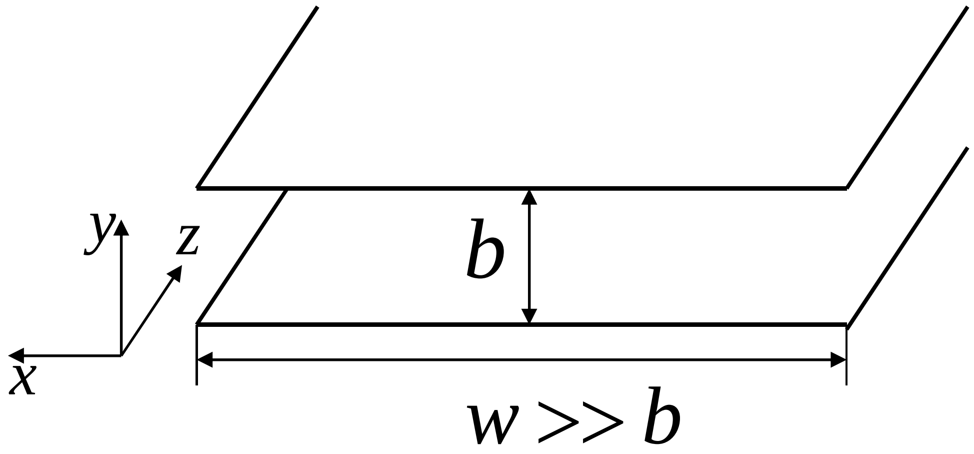
Lect. 11: Metallic Waveguide

Can we send EM waves without worrying about diffraction?

Consider Metallic Waveduide



(From EM2)

TEM Solutions

$$\bar{E} = \bar{y} E_0 e^{-j\beta z}$$

$$\bar{H} = -\bar{x} H_0 e^{-j\beta z}$$

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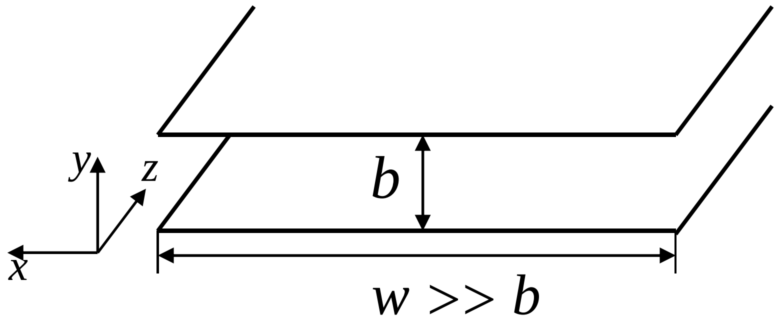

TM $_m$ Solutions (Transverse Magnetic)

$$E_x(y,z)=0$$

$$E_y(y,z)=-\frac{j\beta}{n\pi/b}A_m\cos(\frac{m\pi}{b}y)e^{-j\beta z}$$

$$E_z(y,z)=A_m\sin\left(\frac{m\pi}{b}y\right)e^{-j\beta z}$$

K-vector?

$$H_x(y,z)=A_m\frac{j\omega\varepsilon}{n\pi/b}\cos\left(\frac{m\pi}{b}y\right)e^{-j\beta z}$$

$$H_y(y,z)=0$$

$$H_z(y,z)=0$$

Quantization of β.
Each m corresponds
to one mode.

$$\beta=\sqrt{\omega^2\mu\varepsilon-\left(\frac{m\pi}{b}\right)^2}$$

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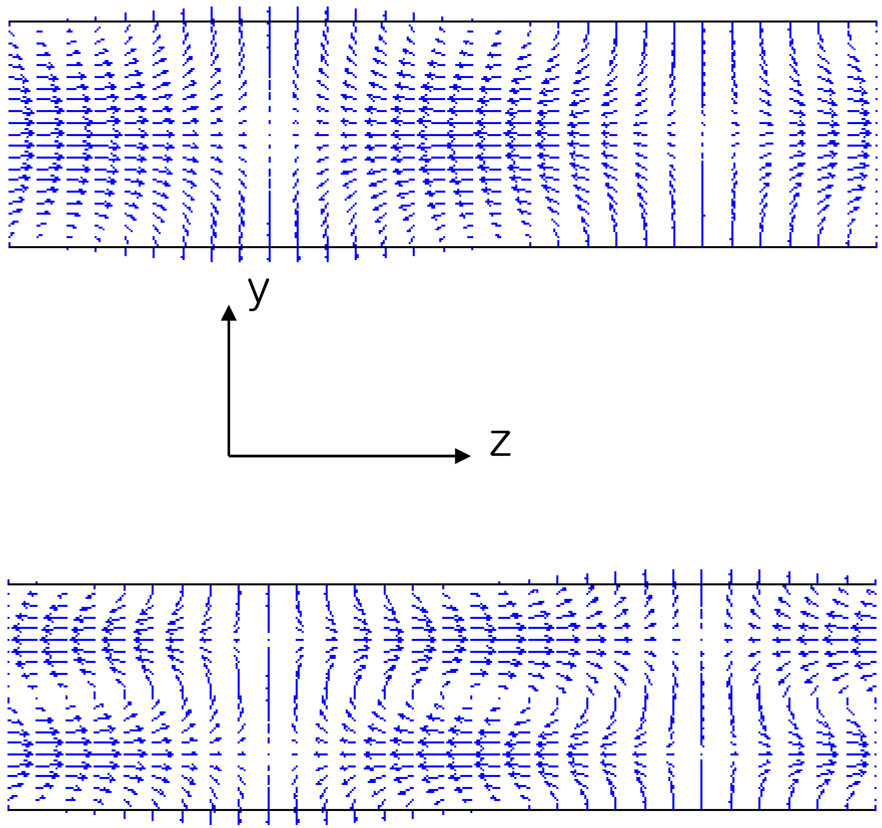
TM$_1$

$$E_z(y,z,t) = A_1 \sin\left(\frac{\pi}{b} y\right) \cos(\omega t - \beta z)$$

$$E_x(y,z,t) = 0$$

$$E_y(y,z,t) = \frac{\beta b}{\pi} A_1 \cos\left(\frac{\pi}{b} y\right) \sin(\omega t - \beta z)$$



TM$_2$

$$E_z(y,z,t) = A_2 \sin\left(\frac{2\pi}{b} y\right) \cos(\omega t - \beta z)$$

$$E_x(y,z,t) = 0$$

$$E_y(y,z,t) = \frac{\beta b}{2\pi} A_2 \cos\left(\frac{2\pi}{b} y\right) \sin(\omega t - \beta z)$$

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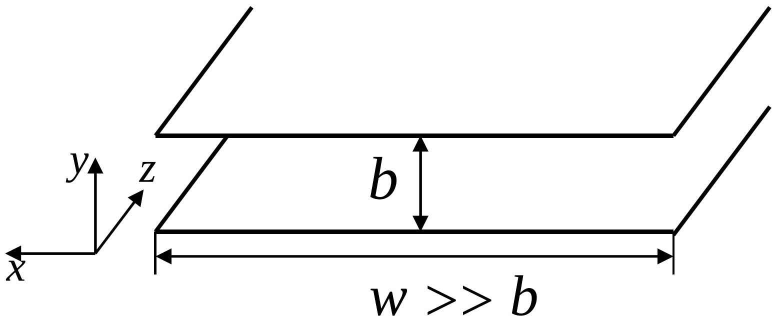


K-vector?

TE$_m$ solutions (Transverse Electric)

$$E_x(y,z) = \frac{j\omega\mu}{m\pi/b} B_m \sin(\frac{m\pi}{b} y) e^{-j\beta z}$$

$$E_y(y,z) = 0$$

$$E_z(y,z) = 0$$

$$H_x(y,z) = 0$$

$$H_y(y,z) = \frac{j\beta}{m\pi/b} B_m \sin(\frac{m\pi}{b} y) e^{-j\beta z}$$

$$H_z(y,z) = B_m \cos(\frac{m\pi}{b} y) e^{-j\beta z}$$

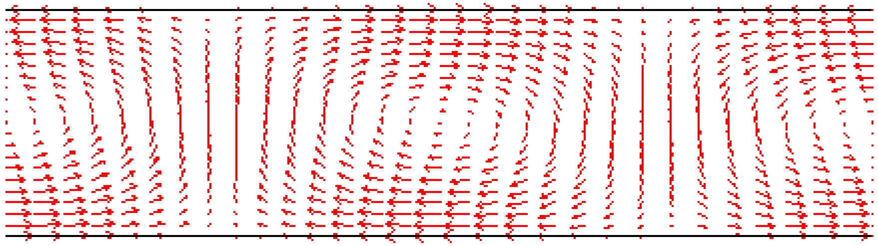
$$\beta = \sqrt{\omega^2 \mu\varepsilon - \left(\frac{m\pi}{b}\right)^2}$$

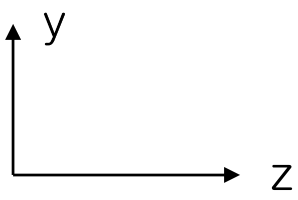
TE$_1$

$$H_x(y,z,t) = 0$$

$$H_y(y,z,t) = -\frac{\beta}{\pi/b} B_1 \sin(\frac{\pi}{b} y)\sin(\omega t - \beta z)$$

$$H_z(y,z,t) = B_1 \cos(\frac{\pi}{b} y)\cos(\omega t - \beta z)$$



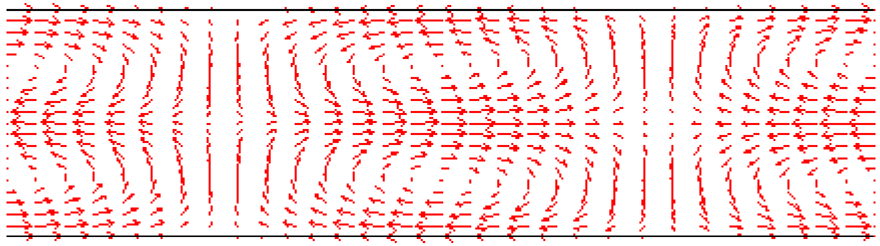


TE$_2$

$$H_x(y) = 0$$

$$H_y(y) = -\frac{\beta}{2\pi/b} B_2 \sin(\frac{2\pi}{b} y)\sin(\omega t - \beta z)$$

$$H_z(y) = B_2 \cos(\frac{2\pi}{b} y)\cos(\omega t - \beta z)$$



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TM and TE difference?

TM$_m$ Solutions

$$E_x(y,z)=0$$

$$E_y(y,z)=-\frac{j\beta}{n\pi/b}A_m\cos(\frac{m\pi}{b}y)e^{-j\beta z}$$

$$E_z(y,z)=A_m\sin\left(\frac{m\pi}{b}y\right)e^{-j\beta z}$$

$$H_x(y,z)=A_m\frac{j\omega\varepsilon}{n\pi/b}\cos\left(\frac{m\pi}{b}y\right)e^{-j\beta z}$$

$$H_y(y,z)=0$$

$$H_z(y,z)=0$$

TE$_m$ solutions

$$E_x(y,z)=\frac{j\omega\mu}{m\pi/b}B_m\sin(\frac{m\pi}{b}y)e^{-j\beta z}$$

$$E_y(y,z)=0$$

$$E_z(y,z)=0$$

$$H_x(y,z)=0$$

$$H_y(y,z)=\frac{j\beta}{m\pi/b}B_m\sin(\frac{m\pi}{b}y)e^{-j\beta z}$$

$$H_z(y,z)=B_m\cos(\frac{m\pi}{b}y)e^{-j\beta z}$$

TM ➔ Parallel Polarization

TE ➔ Perpendicular Polarization

Lect. 11: Metallic Waveguide

- Why sin, cos dependence on y?

$$e^{jk_y y} \pm e^{-jk_y y} \sim \cos(k_y y),\ \sin(k_y y)$$

$$==> k_y = k\cos\theta = \frac{m\pi}{b}$$

- Why quantization in k_y or θ? (➔mode)

In y-direction, wave goes through a periodic movement.

$$e^{-jk_y b}(-1)e^{-jk_y b}(-1) = e^{-j2k_y b} = 1$$

$$\therefore 2k_y b = 2m\pi \text{ and } k_y = \frac{m\pi}{b},\ \beta(=k_z) = \sqrt{(nk_0)^2 - \left(\frac{m\pi}{b}\right)^2}$$

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- How many modes can a given waveguide support?

From $k_y = \dfrac{m\pi}{b}$, $nk_0 \cos\theta = \dfrac{m\pi}{b}$ and $\beta = \sqrt{(nk_0)^2 - \left(\dfrac{m\pi}{b}\right)^2}$

As m increases,

θ decreases until it becomes zero, and β goes down until it become zero.

=> $nk_0 > \dfrac{m\pi}{b}$ $\quad \therefore$ m (integer) $< \dfrac{2nb}{\lambda}$

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- For a given waveguide mode, what is the min. frequency that can propagate?

From $\beta = \sqrt{\omega^2 \mu\varepsilon - \left(\frac{m\pi}{b}\right)^2}$

as ω decreases, β decreases until it becomes zero.

$$=> \omega^2 \mu\varepsilon = \left(\frac{m\pi}{b}\right)^2$$

$$\therefore \omega_{\min} = \frac{m\pi}{b\sqrt{\mu\varepsilon}} \text{ or } f_{\min} = \frac{m}{2b\sqrt{\mu\varepsilon}}; \text{ Cut-off Frequency}$$

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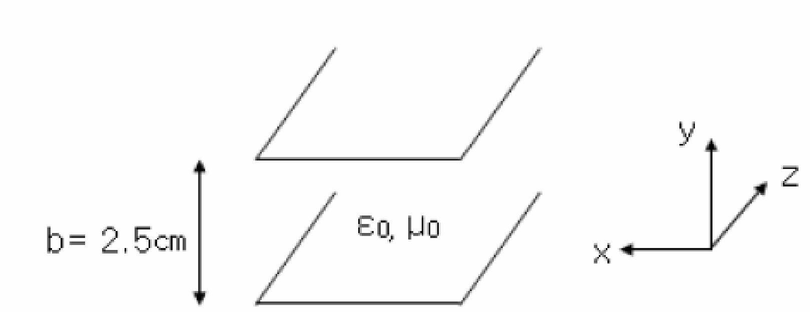
Homework: (전자장 1999 Test 3-1) Due on 10/18

Consider a lossless parallel-plate waveguide shown below.



(a) How many TE or TM modes are there that can propagate in the waveguide if the EM wave frequency is 30 GHz?

(b) Find the expression for H(x,y,z) of TM_2 mode. Assume any proportional constant is 1.

(c) Determine the time-dependent electric charge density distribution on the top plate at z=0 for TM_2 mode.

(d) If we decompose TM_2 mode into plane waves, what is the incident angle in degrees on the top plate that the plane wave has? See below for the definition of the incident angle.